

Warm-Up:

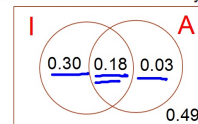
In a recent survey 48% of students said that they own an iPod, 21% of students said they use an Apple computer, and 18% of students said they have both. Construct a Venn diagram:

Find the probability that a randomly selected student:

1. Has an iPod or an Apple computer.
2. Doesn't have an iPod but uses an Apple computer.
3. Doesn't have an iPod or uses an Apple computer
4. Doesn't have either.
5. Has an iPod or doesn't use an Apple computer

Warm-Up:

In a recent survey 48% of students said that they own an iPod, 21% of students said they use an Apple computer, and 18% of students said they have both. Construct a Venn diagram:



Find the probability that a randomly selected student:

1. Has an iPod or an Apple computer. $P(I \text{ or } A) = 0.51$
2. Doesn't have an iPod but uses an Apple computer. $P(I^c \text{ and } A) = 0.03$
3. Doesn't have an iPod or uses an Apple computer. $P(I^c \text{ or } A) = 0.70$
4. Doesn't have either. $P(I^c \text{ and } A^c) = 0.49$
5. Has an iPod or doesn't use an Apple computer. $P(I \text{ or } A^c) = 0.97$

Given that someone owns an iPod, what's the % own an apple? $\frac{0.18}{0.48}$

CONDITIONAL PROBABILITY

* Probability of an event occurring **given** another event happened first.

* Probability of an outcome given a set condition

* Notation: Probability of A given B = $P(A|B)$ $P(\text{Apple} | \text{iPod}) = \frac{0.18}{0.48}$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Example 1:

1. What is the probability that a randomly selected male plays an instrument?

Sex	Do you play an instrument?		
	Yes	No	Total
Male	15	26	41
Female	32	27	59
Total	47	53	100

$$P(I|M) = \frac{15}{41} = 0.366 = 36.6\%$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Sex	Do you play an instrument?		
	Yes	No	Total
Male	15	26	41
Female	32	27	59
Total	47	53	100

2. What is the probability that if a person plays an instrument that they are male?

$$P(M|I) = \frac{15}{47} =$$

3. What is the probability that a person is female given that they play an instrument?

$$P(F|I) = \frac{32}{47} =$$

4. We stop a male in the hallway at school. What is the chance they do not play an instrument?

$$P(N|M) = \frac{26}{41}$$

Example 2:

	High School	2 yr college	4+ yr college	Totals
Smoker	32	5	13	50
Non-Smoker	61	17	72	150
Totals	93	22	85	200

1. What is the probability that a person smokes?

$$P(S) = \frac{50}{200} =$$

2. What is the probability that a person that smokes had a high school degree only?

$$P(H|S) = \frac{32}{50}$$

3. What is the probability that a person that had a high school degree only was a smoker?

$$P(S|H) = \frac{32}{93}$$

4. What is the chance that a person smokes given they had a 2 year college degree?

$$P(S|2) = \frac{5}{22}$$

5. What is the probability that if a person had a 4+ year college degree, that they were a smoker?

$$P(S|4) = \frac{13}{85}$$

6. What is the probability that a nonsmoker had a 4+ year college degree?

$$P(4|N) = \frac{72}{150}$$

7. What is the probability that a smoker had a 4+ year college degree?

$$P(4|S) = \frac{13}{50}$$

Example #3

A survey of 180 summer campers compared the sex of the camper to whether they suffered from any allergies.

	Female	Male	Total
Allergy	18	27	45
No Allergy	54	81	135
Total	72	108	180

1. What is the probability that a child has an allergy?

$$P(A) = \frac{45}{180}$$

2. What is the probability that a male has an allergy?

$$P(A|M) = \frac{27}{108}$$

3. What is the probability that a female has an allergy?

$$P(A|F) = \frac{18}{72}$$

	Female	Male	Total
Allergy	18	27	45
No Allergy	54	81	135
Total	72	108	180

4. What is the probability that a child is male, given they don't have an allergy?

$$P(M|A^c) = \frac{81}{135}$$

5. What is the probability that a child is female, given they don't have an allergy?

$$P(F|A^c) = \frac{54}{135}$$

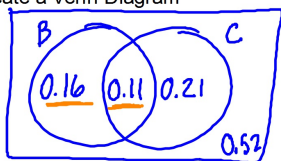
6. What is the probability that an allergy sufferer is a female?

$$P(F|A) = \frac{18}{45}$$

7. What is the probability that if the child is male, they don't have an allergy?

$$P(A^c|M) = \frac{81}{108}$$

Example 4: The probability that an American Male has High Blood Pressure is 27%, High Cholesterol is 32%, and both High Blood Pressure and High Cholesterol is 11%. Create a Venn Diagram



1. What is the probability that a man has high blood pressure given that he has high cholesterol?

$$P(B|C) = \frac{0.11}{0.32} = 0.34375$$

2. What is the probability that a man with high blood pressure has high cholesterol?

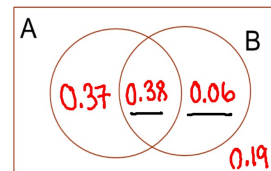
$$P(C|B) = \frac{0.11}{0.27} = 0.4074$$

Example 5:

$$P(A) = 0.75$$

$$P(B) = 0.44$$

$$P(A \text{ and } B) = 0.38$$



$$1. P(A|B) = \frac{0.38}{0.44} = 0.8636$$

$$2. P(B|A) = \frac{0.38}{0.75} = 0.5067$$

$$3. P(A|\text{not } B) = \frac{0.37}{0.56} = 0.6607$$

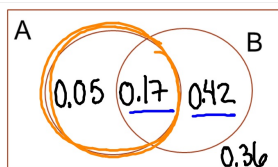
$$4. P(\text{not } A|B) = \frac{0.06}{0.44} = 0.1364$$

Example 6:

$$P(A) = 0.22$$

$$P(B) = 0.59$$

$$P(A \text{ and } B) = 0.17$$



$$1. P(A|B) = \frac{0.17}{0.59} = 0.288$$

$$2. P(B|A) = \frac{0.17}{0.22} = 0.773$$

$$3. P(A|\text{not } B) = \frac{0.05}{0.41} = 0.122$$

$$4. P(\text{not } A|\text{not } B) = \frac{0.36}{0.41} = 0.878$$

$$5. P(\text{not } A|B) = \frac{0.42}{0.59} = 0.712$$

Example 7: Suppose you have a bag of skittles with 12 red, 8 blue, 7 green, 9 yellow, and 6 orange candies. **total = 42**

1. What is the probability that you would randomly select a green skittle?

$$\frac{7}{42}$$

2. What is the probability that if you had already eaten a red skittle that you would randomly select a green skittle?

$$\frac{7}{41}$$

3. If you had eaten 5 of the green in a row, what would be the probability that the next skittle would be green?

$$\frac{2}{37}$$

4. If you had eaten 8 red in a row, what would be the probability that the next skittle would be green?

$$\frac{7}{34}$$

Example 8: Rolling a standard die.

1. What is the probability that you would roll a 5? $\frac{1}{6}$
2. If you had just rolled a 3, what is the probability that you would roll a 5? $\frac{1}{6}$
3. If you rolled a 5 six times in a row, what is the probability that the next roll would be 5? $\frac{1}{6}$

Independence:

* Two events are independent if the occurrence of one event doesn't have an effect on the chance of the other event.

opposite = dependent

* If A and B are independent then:

$$P(A|B) = P(A)$$

$$P(5|6) = P(5)$$

* Note: If two events are disjoint (mutually exclusive), then they cannot be independent

$$P(I) = P(I|F)$$

Example 1: Is being Male and Playing an instrument independent?

$$P(M) = \frac{41}{100} = 0.41$$

$$P(M|I) = \frac{15}{47} = 0.319$$

$$P(F) = P(F|N)$$

Sex	Do you play an instrument?		Total
	Yes	No	
Male	15	26	41
Female	32	27	59
Total	47	53	100

Example #2:

Are smoking and having a high school degree independent?

$$P(S) = \frac{50}{200} = 25\%$$

$$P(S|H) = \frac{32}{93} = 34.4\%$$

	High School	2 yr college	4+ yr college	Totals
Smoker	32	5	13	50
Non-Smoker	61	17	72	150
Totals	93	22	85	200

No, not indep.

Example #3: Is sex and having an allergy independent?

$$P(F) = P(F|A)$$

$$\frac{72}{180} = \frac{18}{45}$$

$$40\% = 40\%$$

Within 5%

	Female	Male	Total
Allergy	18	27	45
No Allergy	54	81	135
Total	72	108	180

$$P(A) = P(A|F)$$

Yes, indep.

Example #4

Are having high blood pressure and high cholesterol independent?

The probability that an American Male has High Blood Pressure is 27%, High Cholesterol is 32%, and both High Blood Pressure and High Cholesterol is 11%.

$$P(C) = P(C|B)$$

$$0.32 = \frac{0.11}{0.27}$$

$$0.32 \neq 0.407$$

No, not indep.



EXAMPLE 5:

Sex	Pierced Ears		Total
	Yes	No	
Male	19	71	90
Female	84	4	88
Total	103	75	178

- What is the probability that a randomly selected student is male?
- What is the probability that a randomly selected student has pierced ears?
- What is the probability that a randomly selected student is male and has pierced ears?
- What is the probability that a randomly selected student has pierced ears given the student is male?
- What is the probability that a randomly selected student is male given the student has pierced ears?

Sex	Pierced Ears		Total
	Yes	No	
Male	19	71	90
Female	84	4	88
Total	103	75	178

- What is the probability that a female has pierced ears?
- What is the probability that a student with pierced ears is female?
- What is the probability that if a student doesn't have pierced ears that they are male?
- What is the probability that if a student doesn't have pierced ears that they are female?
- Are gender and pierced ears independent?

- a. What is the probability that a randomly selected student is male?

$$P(M) = \frac{90}{178} = 0.5056$$

- b. What is the probability that a randomly selected student has pierced ears?

$$P(P) = \frac{103}{178} = 0.5787$$

- c. What is the probability that a randomly selected student is male and has pierced ears?

$$P(M \text{ and } P) = \frac{19}{178} = 0.1067$$

- d. What is the probability that a randomly selected student has pierced ears given the student is male?

$$P(P|M) = \frac{19}{90} = 0.2111$$

- e. What is the probability that a randomly selected student is male given the student has pierced ears?

$$P(M|P) = \frac{19}{103} = 0.1845$$

- f. What is the probability that a female has pierced ears?

$$P(P|F) = \frac{84}{88} = 0.9545$$

- g. What is the probability that a student with pierced ears is female?

$$P(F|P) = \frac{84}{103} = 0.8155$$

- h. What is the probability that if a student doesn't have pierced ears that they are male?

$$P(M|not P) = \frac{71}{75} = 0.9467$$

- i. What is the probability that if a student doesn't have pierced ears that they are female?

$$P(F|not P) = \frac{4}{75} = 0.0533$$

- j. Are gender and pierced ears independent?

$$P(P|F) = 0.9545 \quad P(P) = 0.5787$$

NO!

Gene	Developed Cancer		
	Yes	No	Total
Yes	23	15	38
No	48	58	106
Total	71	73	144

EXAMPLE 6

A research study was conducted to see if having a specific gene increased the chance that a person developed cancer. The results are summarized in the following table.

- What is the probability someone developed cancer?
- What is the probability that someone had the gene?
- What is the probability that someone had the gene and developed cancer?
- What is the probability that someone with the gene developed cancer?
- What is the probability that someone with cancer had the gene?

Gene	Developed Cancer		
	Yes	No	Total
Yes	23	15	38
No	48	58	106
Total	71	73	144

- f. What is the probability that someone that didn't have the gene developed cancer?

- g. What is the probability that someone didn't have the gene given they had cancer?

- h. What is the probability that someone didn't have cancer given they had the gene?

- i. Are GENE and CANCER independent?

- a. What is the probability someone developed cancer?

$$P(C) = \frac{71}{144} = 0.4931$$

- b. What is the probability that someone had the gene?

$$P(G) = \frac{38}{144} = 0.2639$$

- c. What is the probability that someone had the gene and developed cancer?

$$P(G \text{ and } C) = \frac{23}{144} = 0.1597$$

- d. What is the probability that someone with the gene developed cancer?

$$P(C|G) = \frac{23}{38} = 0.6053$$

- e. What is the probability that someone with cancer had the gene?

$$P(G|C) = \frac{23}{71} = 0.3239$$

- f. What is the probability that someone that didn't have the gene developed cancer?

$$P(C|not G) = \frac{48}{106} = 0.4528$$

- g. What is the probability that someone didn't have the gene given they had cancer?

$$P(not G|C) = 48/71 = 0.6761$$

- h. What is the probability that someone didn't have cancer given they had the gene?

$$P(not C|G) = 15/38 = 0.3947$$

- i. Are Gene and Cancer independent?

$$P(C|G) = 0.6053$$

$$P(C) = 0.4931$$

NO!